Adjoint Optimization of Wind Farm Layouts for Systems Engineering Analysis

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Wind is an important renewable energy resource, but in order for it to be competitive with other energy sources, wind farm layouts must be optimized to reduce the levelized cost of energy (LCOE). This requires an analysis that integrates high-fidelity flow models with physical turbine and cost models. We present a new flow model and turbine optimization tool, WindSE, that optimizes wind turbine locations and axial induction factors as part of the National Renewable Energy Laboratory’s open source wind energy systems engineering software tool WISDEM. WindSE enables gradient-based layout optimization by solving the discrete adjoint equations corresponding to the forward flow model. This provides a computationally efficient way to calculate gradients used in the optimization process. We discuss the development of WindSE and provide optimization results based on simulations of single and multiple inflow directions. In the case of strongly directional wind roses the optimal layouts take advantage of flow curvature and speedup effects that are not generated by standard industry linear flow models. When optimizing with respect to a uniform wind rose, local speedup effects are found to be less beneficial and the optimal layouts instead reduce wake losses by maximizing spacing. Sensitivity to wind direction bin sizes and weights is demonstrated with a full layout and axial induction factor optimization using a wind rose from an offshore wind farm.

I. Introduction

Optimizing wind turbine locations within a wind farm is a difficult problem due to the strong nonlinear coupling between turbine locations, power production, atmospheric boundary layer turbulence, and mechanical loads on turbine components. Existing wind farm optimization techniques used by industry often rely on heuristic guidelines and simplified linear flow models. Traditional approaches to optimizing turbine layouts have focused on maximizing total power output from linear flow models, but this does not take into account the costs of constructing, operating, and maintaining turbines within a farm. If not properly assessed, these...
costs can become prohibitive even for wind farms where power output is maximized, thereby limiting the economic competitiveness of wind energy compared to other renewable and non-renewable sources of electricity. Recent studies\textsuperscript{1,2} have also used higher fidelity flow models to find optimal turbine spacing or control settings in an operational context, but a full layout optimization that considers multiple inflow directions has not yet been attempted.

In order to develop a more rigorous approach to wind turbine layout optimization, a systems engineering analysis that considers the Levelized Cost of Energy (LCOE) with high fidelity simulations is required. With an LCOE-based optimization routine, tradeoffs between power production, operations and maintenance (O&M) costs, and construction costs can be quantified, thus helping developers determine the most economically competitive wind farm designs. A key challenge in using LCOE as an optimization metric is in estimating O&M costs. These costs are due in large part to damage-equivalent loads on mechanical components due to atmospheric turbulence. These loads are strongly coupled to unsteady coherent turbulence caused by wake interactions and atmospheric instability, and their accurate prediction places enormous demands on the spatial and temporal resolution of flow field data used in the optimization procedure. Furthermore, real wind farms experience inflow from practically all directions but often have one or two primary inflow directions. Capturing the correct distribution of wind directions in the flow modeling is crucial to optimizing the power production and structural loads of a particular layout.

To address these challenges, we present WindSE, a wind farm layout optimizer that uses high fidelity flow models and gradient-based optimization enabled by adjoint techniques. The layout optimization software is part of the National Renewable Energy Laboratory open source wind energy systems engineering analysis package WISDEM.\textsuperscript{3} WISDEM offers a modular framework that allows users to interchange different models for the physical turbine, capital costs, O&M costs, balance of station (BOS) costs, and the site flow model. This facilitates multi-fidelity LCOE-based analysis of wind farm designs that accounts for multi-directional inflow.

The present paper is organized as follows: a background on wind farm layout optimization and adjoint techniques is provided in Section II, the specific wind farm optimization problem, constraints, and turbine representation are described in Section III, details of the numerical implementation of WindSE are covered in Section IV, the analysis and optimization capabilities of WindSE are demonstrated on several test cases with increasingly complex wind roses in Section V, and we discuss our results and plans for continued development in Section VI.

II. Wind Farm Design and Optimization Background

Utility scale wind farms in the U.S. typically involve tens to hundreds of turbines arranged in semi-regular arrays. The layout topology is generally an outcome of optimizing the power production subject to competing influences from site constraints. These constraints include the patchwork of viable building areas formed by leases and setbacks from environmental concerns or physical infrastructure, terrain and soil characteristics like slope or vegetation, turbine manufacturer spacing requirements, and continuity requirements imposed by access roads and electrical connections. This results in a complex design problem with turbine layouts varying drastically between different geographic locations and exhibiting complex topologies.

The annual energy production (AEP) from a wind farm layout has traditionally been, and generally continues to be, assessed using reduced order engineering wind flow models. These models estimate the relative wind resource across a site based on the linearized Navier-Stokes equations and treat terrain features as perturbations in boundary conditions. The underlying governing equations are based on analytical perturbation solutions to flow over a low hill introduced by Jackson and Hunt\textsuperscript{4} in 1975 and implemented in packages like MS3DJH/3R.\textsuperscript{5} This approach decomposes the terrain into sinusoidal hills and calculates relative speedup effects over each hill. The velocity deficit from a single turbine is then superimposed on the background wind resource at each turbine location. The PARK model developed by Jensen\textsuperscript{6} and the eddy viscosity model developed by Ainslie\textsuperscript{7} are the most common models used. These approaches decouple the wake calculation from the wind flow calculation, and further assume that the wake from a single turbine in isolation is appropriate for all turbines in a utility scale farm. Since these models are relatively inexpensive to run, they are amenable to gradient-free optimization techniques, such as genetic algorithms, which can require orders of magnitude more iterations than gradient-based optimization techniques. However, the linear superposition principles that underpin this approach result in poor accuracy in complex terrain, complex flow conditions, or farms with many rows of turbines.
A. Past and Present Approaches to Wind Farm Optimization

Wind farm layout optimization has been performed to date primarily using engineering flow models with analytical wake deficits. These models are inexpensive to run but lack the fidelity to accurately capture turbulent flow effects. Their relatively cheap computational cost allows gradient-free methods to be used in the optimization process. Studies utilizing a gradient-free approach have looked at layouts that minimize wake loss, maximize net present value, and minimize noise propagation. Other optimization approaches include particle swarm optimizations that determine turbine layouts and rotor diameters, extended pattern searches for multimodal layout optimization, and even game theoretic methods. Fleming et al. used finite difference gradients with engineering wake models in a recent optimization study. An exhaustive review of wind farm optimization efforts has been compiled by Herbet-Acero et al. The results of these optimizations are heavily influenced by the analytical wake models, and consequently we expect significant differences in the optimal layout when higher fidelity flow models are used.

Recently, higher fidelity flow models have been used in a limited range of wind farm optimization applications. The Technical University of Denmark (DTU) has developed TOPFARM which employs an improved wake model, the dynamic wake meandering model, as well as a parabolic Navier-Stokes solver. TOPFARM employs a hybrid optimization approach that combines sequential linear programming (SLP) with gradient-free genetic algorithms. The genetic algorithm is used to find the neighborhood of the global optimum, then the gradient-based SLP algorithm takes over. The genetic algorithm step still penalizes large design spaces, limiting TOPFARM to relatively small layouts.

Large eddy simulations (LES) are also increasingly being applied to wind farm modeling and in some cases for layout optimization. Meneveau and collaborators have extensively studied the effects of spacing and alignment in infinite wind farms with turbines in regular gridded turbine arrays. Their studies are based on pseudospectral LES that captures many of the most important atmospheric boundary layer turbulence effects and complex wake interactions, but consider only the spacing between predefined structured rows in their optimization. Goit and Meyers have used similar LES methods with adjoint techniques to study optimal wind farm control by adjusting rotor properties in an operational context. This benefits from high fidelity fluid dynamics and leverages the power of adjoint optimization, but once again the optimization has been simplified by using fixed layouts. Funke et al. studied adjoint optimization of ocean turbine layouts, but they used the shallow water equations making their results incompatible with atmospheric boundary layer flow fields.

The present approach is to use adjoint optimization techniques to enable gradient-based optimization of wind turbine layouts in realistic atmospheric boundary layer flows. Laminar flow, Reynolds Averaged Navier-Stokes (RANS), and LES solvers are being developed that can be used interchangeably in the WISDEM framework. This allows for efficient, high fidelity layout optimization capabilities that do not currently exist. Furthermore, we emphasize optimization in the presence of multi-directional inflow angles that better reflect real wind farm atmospheric conditions.

B. Adjoint Optimization Enables High Fidelity Flow Models

It is well known that engineering models based on simplifications of the governing equations are inaccurate in wind farms with complex terrain, non-neutral atmospheric stability, and multiple turbine rows. High fidelity flow models that account for terrain, stability, and turbulence are necessary to improve the accuracy of AEP and loading predictions. These models incur greater computational costs than existing engineering models, which makes gradient-free optimization approaches intractable. Instead, gradient-based optimizations are necessary to reduce the number of iterations and expensive function evaluations. A conventional finite difference approach to calculating the necessary gradients is also prohibitively expensive because it requires a function evaluation for each design variable. For utility scale wind farms with several hundred position variables this negates many of the advantages of gradient-based optimization.

Fortunately, the necessary gradients can be obtained relatively inexpensively using adjoint optimization techniques. An adjoint approach allows one to calculate gradients at a cost that scales with the number of objective functions rather than the number of design variables. For wind farm optimization with a single LCOE objective function this means that gradients can be found by solving the adjoint or dual partial differential equation (PDE) at roughly the same cost as a single forward model function evaluation.

Adjoint techniques have a rich history in fluid dynamics. They were been used to evaluate uncertainty and errors with the intent
of improving meshes. Adjoint techniques for PDE constrained optimization were popularized by Lions, who later expanded to flow control by Pironneau and finally adapted to fluid dynamics by Jameson, who applied control theory to airfoil shape optimization. Giles and Pierce put the adjoint approach on a solid mathematical foundation by investigating the behavior of boundary conditions, quasi-1D Euler equations, shocks, and various formulations of the continuous adjoint. Reviews of implementation of discrete and continuous adjoints in optimization practices have been published by Jameson, Giles and Pierce, and Giannakoglou and Papadimitriou.

Adjoint techniques have seen uses in many other fields and applications, including geophysical flows such as mantle convection, numerical weather prediction and data assimilation. Other applications include mesh optimization, error and uncertainty analysis, and large scale multi-disciplinary analysis and optimization. Investigation of unsteady adjoints of chaotic systems by Wang and Blonigan shows a strong sensitivity to initial conditions, numerical accuracy, and important dynamical properties. This makes finding adjoint gradients of chaotic PDEs, like the LES equations, very challenging.

C. Background on Adjoint Optimization Techniques

Adjoint operators, adjoint equations, and adjoint variables arise naturally from consideration of dynamical systems whose behavior can be described by differential equations. All that is required is an ordinary or partial differential equation governing the evolution of a dynamical system and a function of the output that measures a quantity of interest called an objective function. The forward operator is the differential equation that takes independent control variables as inputs and advances the state variables forward in time. The adjoint operator maps perturbations in the input variables to gradients of the objective function. These adjoint variables are, in a sense, optimal open-loop control inputs for the given objective function.

The adjoint operator is defined using an inner product by the bilinear identity where is adjoint to . This holds if is a continuous differential operator, in which case is found through integration by parts, or if is a matrix, in which case , the conjugate transpose of . This bilinear identity reveals that the adjoint operator is implicitly defined by the forward model and can achieve the same scalar output as but with a different order and size of matrix-vector multiplications. Fundamentally, it is this change in the order and dimension of matrix-vector multiplications that lends adjoint methods their computational power. In the following, we will show how utilizing this identity while calculating gradients of an objective function achieves dramatic computational savings. Due to changes in the size and order of matrix-vector products inherent in the adjoint process, the computational cost of finding gradients scales with the dimension of the cost function, which is typically just a scalar, instead of with the dimension of the design variable space.

Consider a dynamical system with governing equations that can be expressed in residual form

\[ F(u, m) \equiv 0 \quad (1) \]

where is a vector-valued differential equation, is the system state vector, and is a vector of input parameters. Additionally, consider a scalar objective functional that measures a quantity of interest. Many engineering problems can be formulated as optimization problems that seek optimal control parameters to minimize \( J \):

\[
\begin{align*}
\text{minimize} & \quad J(u, m) \\
\text{subject to} & \quad F(u, m) = 0 \\
& \quad h(m) = 0 \\
& \quad g(m) \leq 0
\end{align*}
\]

where and are equality and inequality constraints on the control parameter , such as upper and lower bounds on a control input. In common engineering problems, is a PDE that is expensive to evaluate and the dimension of the control space is high. For example, could contain the initial conditions for a numerical weather prediction model with being the forward operator of the weather model, or might describe the shape of an airfoil that is being optimized for lift subject to the Euler equations in . Efficiently solving this PDE-constrained optimization problem requires algorithms that scale well to high dimensions and minimize the number of evaluations of . Gradient-based optimization algorithms can approach second order convergence to local minima and can minimize the evaluations of , but require the gradient of the
objective functional with respect to all of the control parameters. This gradient, \( \frac{dJ}{dm} \), is given by the chain rule as

\[
\frac{dJ}{dm} = \frac{\partial J}{\partial m} + \frac{\partial J}{\partial u} \frac{\partial u}{\partial m}.
\]  

(3)

Since \( J(u,m) \) is a user defined function, \( \frac{\partial J}{\partial m} \) and \( \frac{\partial J}{\partial u} \) are straightforward to compute. However, taking \( \frac{\partial}{\partial m} (\cdot) \) to be a row vector, \( \frac{\partial u}{\partial m} \in \mathbb{R}^{n \times m} \), which makes \( \frac{\partial u}{\partial m} \) very difficult to compute for high dimensional control and state spaces. A finite difference approach to calculate this gradient would require \( m \) evaluations of \( F \), which is intractable for many contemporary engineering problems.

In the adjoint approach, \( \frac{\partial u}{\partial m} \in \mathbb{R}^{n \times m} \) is eliminated from Eq. (3) by taking the derivative of the PDE constraint \( F(u,m) = 0 \):

\[
\frac{\partial F}{\partial m} + \frac{\partial F}{\partial u} \frac{\partial u}{\partial m} = 0.
\]  

(4)

Solving for \( \frac{\partial u}{\partial m} \) and substituting into Eq. (3) yields:

\[
\frac{dJ}{dm} = \frac{\partial J}{\partial m} - \frac{\partial J}{\partial u} [\frac{\partial F}{\partial u}]^{-1} \frac{\partial F}{\partial m} \Psi^T
\]  

(5)

where \( \Psi \) is the adjoint variable and maps source perturbations in \( F(u,m) = 0 \) to sensitivities of \( J \). It is governed by:

\[
[\frac{\partial F}{\partial u}]^T \Psi = [\frac{\partial J}{\partial u}]^T.
\]  

(6)

With a forward solution of \( F \) and the adjoint solution \( \Psi \), the derivative of the objective function can be readily calculated as

\[
\frac{dJ}{dm} = \frac{\partial J}{\partial m} - \Psi^T \frac{\partial F}{\partial m}.
\]  

(7)

The resulting adjoint gradients are typically more accurate than finite difference gradients, and can be calculated at a fixed cost that is independent of the dimension of \( m \). This enables efficient gradient-based optimization for systems governed by computationally demanding PDEs.

### III. Wind Farm Optimization Problem

Wind farm layout optimization is approached as a PDE-constrained optimization problem using the adjoint gradient theory developed in the previous section. The PDE constraint \( F \) describes fluid flow within a wind farm, the turbine locations are the control vector \( m \), and turbine positioning rules constrain the possible locations.

#### A. Optimization Problem Definition

The long term vision for the WISDEM software is to implement an objective functional that calculates LCOE based on energy production, O&M costs, and turbine capital costs. The WindSE model is responsible for the energy production portion, and for the purposes of model testing we focus solely on an energy production optimization. We seek to maximize the steady state power output of \( N \) turbines experiencing \( K \) different
inflow directions by posing the following optimization problem:

\[
\min_{m=[x,y,a]} J = - \sum_{m=1}^{K} \sum_{n=1}^{N} \alpha_k \frac{1}{2} \rho A C_{p,n} \| u(x_n, y_n) \|^3
\]  

subject to

\[
\frac{\partial u_i}{\partial x_j} = - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + \frac{1}{\rho} \sum_{n=1}^{N} f_{n,AD}
\]  

\[
\frac{\partial u_i}{\partial x_i} = 0
\]  

\[
f_{n,AD} = \frac{1}{2} \rho A C_{t,n} \| u(x_n, y_n) \|^2
\]  

\[
C_{p,n} = 4 \delta_n (1 - a_n)^2
\]  

\[
C_{t,n} = 4 \delta_n (1 - a_n)
\]  

\[
L_x < x_n < U_x
\]  

\[
L_y < y_n < U_y
\]  

where the subscript \( n \) indexes the individual turbines, the subscript \( k \) indexes the different flow directions, \( \alpha_k \) is a weight factor for a given inflow direction, \( A \) is the turbine rotor area, \( C_p \) is the turbine power coefficient, \( C_t \) is the turbine thrust coefficient, \( u \) is the simulation flow field, and \((x_n, y_n)\) is the position of turbine \( n \). The body force from each turbine \( f_{n,AD} \) is calculated based on the velocity at the center of the rotor and the force is distributed across the rotor plane. The turbines are constrained to remain within a rectangular box described by the upper and lower bounds in the \( x \) and \( y \) directions given by \( L_x, L_y, U_x, U_y \). For our initial study we focus on optimizing \( J \) by finding optimal turbine positions and axial induction factors represented by the control vector \( m = [x, y, a] \) for \( n = 1 \ldots N \).

B. Governing Equations

WindSE is being developed to have laminar, RANS, and LES flow solver capabilities. Our initial results are for low Reynolds number steady state laminar flow conditions. The governing equations in this case are simply the incompressible steady state Navier Stokes equations with an \( \mathcal{O}(100) \) Reynolds number and a body force imparted by the turbine actuator disk. This is given by Eqs. (9) and (10) with \( u_i \) the laminar velocity and \( \nu \) the kinematic viscosity. The viscosity in the implemented form of the equations can be readily augmented with an eddy viscosity to incorporate a RANS turbulent flow solver.

C. Turbine Representation

The turbines are represented as non-rotating actuator disks using standard actuator disk theory. When not optimizing the axial induction factor, we assume a constant axial induction factor \( a = \frac{1}{3} \) and calculate the body force from turbine \( n \) according to Eq. (11). The turbine power production is calculated based off the rotor velocity according the formulation in Eq. (8). The turbine body force is smoothed over the rotor disk in the streamwise and spanwise directions using a Gaussian distribution.

D. Initial and Boundary Conditions

Dirichlet boundary conditions are used to prescribe the velocity and wind direction on inflow boundaries. The pressure is set to 0 on outflow boundaries. A steady state solution is found for each of the \( K \) inflow directions. Different inflow directions are achieved by applying a rotation to the turbine coordinates \( x \) and \( y \). The rotation is incorporated into the adjoint gradient calculation and allows the same computational domain to be used for the entire wind rose.
IV. Solution Implementation

A. WindSE Numerical Solver

The WindSE flow solver is implemented in a software package called FEniCS,\textsuperscript{49} which automates the solution of PDEs using the finite element method. FEniCS is written in Python, and can be easily integrated with other systems engineering tools like WISDEM. FEniCS comes with a user-friendly problem solving environment with a mathematical syntax for writing the variational form of PDE problems and meshing the finite element, and automated compilers for generating finite element basis functions and assembling the variational forms. The Python code is just-in-time (JIT) compiled to C++ for computational speed. FEniCS can interface to common HPC libraries such as PETSc and Trilinos for numerical linear algebra, ParMETIS and SCOTCH for domain decomposition, and MPI and OpenMP for parallelization. FEniCS has been well tested and validated on a number of computational problems in solid and fluid dynamics, eigenvalue problems, and coupled PDEs.

The variational form of finite element problems lends itself well to gradient based optimization problems. The software package dolfin-adjoint\textsuperscript{50} performs a high level automated differentiation and can derive both discrete adjoint and tangent linear models of a forward problem implemented in FEniCS. The discrete adjoint and tangent linear models are important in the gradient based optimization algorithms used in data assimilation, optimal control, and error estimation. The automatic differentiation routines in dolfin-adjoint treat each forward model as a series of equation solves and therefore derives adjoint and tangent linear models at a higher level of abstraction than traditional algorithmic differentiation, which treats forward models as a series of elementary instructions. This higher level of abstraction gives dolfin-adjoint greater flexibility and automation across a wide range of PDE applications. Moreover, dolfin-adjoint can be implemented on unsteady and nonlinear PDEs, and can also be run in parallel. It can directly interface to the optimization algorithms in SciPy and also contains routines for checking the correctness of adjoint gradients.

We solve Eqs. (9) and (10) in a coupled fashion using a Taylor-Hood mixed finite element space. This involves first order Lagrange elements for the pressure field and second order Lagrange elements for the velocity field and has favorable built-in stability properties. The equations are solved using a nonlinear Newton solver with Jacobians calculated using the automatic differentiation tools available in FEniCS.

B. Gradient Based Optimization Process

We use gradient-based algorithms implemented in Python’s SciPy module to find local minima using the adjoint gradients provided by dolfin-adjoint. Specifically, the BFGS and SLSQP algorithms are used depending on the type of constraint. Both algorithms allow for bounds on turbine locations, but SLSQP is required for implementing constraints on spacing between turbines.

Since the optimization algorithms are gradient-based, they find local rather than global minima. The layout optimization problem can have many local minima, particularly with few turbines and few inflow directions. We take a statistical approach to finding the global optimum and sample many different random starting positions. We note that with many inflow directions, the optimization problem actually becomes more convex and fewer optimization iterations are required to find minima.

The optimizations were run on the NREL high performance computer Peregrine and were parallelized using MPI. The discrete adjoint calculation is automatically parallelized by dolfin-adjoint if the forward model is run in parallel, drastically simplifying code development. The optimization typically required 50 – 75 iterations to converge to an optimum for a single flow direction. There are more function evaluations than optimization iterations because the line search method used to determine step size at each iteration often requires several function evaluations. In multi-direction optimizations the forward runs are more expensive because many inflow directions must be evaluated, the optimization is more convex and typically converges in less than 30 iterations. Figure 1 shows a schematic of the multi-direction optimization workflow. The layout and axial induction factors are optimized over all inflow directions simultaneously in a multilevel optimization process rather than in a sequential optimization process.
V. Optimization Results For Single and Multiple Inflow Directions

We present results for turbine layout and axial induction factor optimization with single and multiple inflow directions. We examine different spacing constraints and find that the optimizer takes advantage of flow curvature effects to capture local speedups in the single inflow direction case. The turbines are generally arranged in a duct that concentrates the flow on downstream turbines. As more directions are considered, the optimal layouts tend to form radially-oriented ducts with even weighting of inflow directions. Weighting a particular direction heavily recovers the single inflow direction solution. We also note that optimal layouts are typically symmetric, providing a useful heuristic to identify whether a solution is caught in a local minimum.

A. Single Inflow Direction Optimization

In the first set of simulations, constant inflow direction and velocity are examined. The turbines are initialized to a regular grid and fixed axial induction factors are used. The optimal layouts and resulting flow fields are shown in Figure 2a and Figure 2b for 16 and 32 turbines, without spacing constraints. The flow is left to right with slip boundary conditions on the top and bottom walls. The turbines are constrained to lie within a rectangular site constraint, as indicated by the black rectangle. Because the adjoint gradient technique finds gradients at a cost that is independent of the number of design variables, the 16 and 32 turbine optimizations involve the same computational expense.

We also explore the effects of turbine spacing constraints by imposing 1 and 1.5 rotor diameter spacing (RD) constraints. This site and minimum turbine spacing is smaller than is typical on real world wind farms, but the results still demonstrate many interesting insights not found in linear flow model optimization results. Figures 3a and 3b show the optimization results for the same simulations as in the unconstrained case. Although the turbines cannot pack as closely together, they still form a duct shape.

The optimized layouts exhibit several noteworthy characteristics. First, the layouts appear to form ducts that accelerate the incoming flow and channel that flow toward downstream turbine rows. Turbines are also placed along the spanwise site constraint walls to keep the flow from diverting around the downstream rows. A similar pattern was observed in the OpenTidalFarm results.21 Another interesting feature is that the downstream turbines are not pushed all of the way to the downstream site constraint. This is a different behavior from that typically observed in linear flow optimization results. In particular, engineering models
Figure 2: Optimization results for a single flow direction left to right. Top row shows optimized turbine locations and flow streamlines, with the black rectangle indicating the site constraint. Bottom row shows the steady state velocity field with red indicating high speeds and blue indicating low speeds.

Figure 3: Optimization results for a single flow direction left to right with 32 turbines and different turbine spacing constraints. Top row shows optimized turbine locations and flow streamlines, with the black rectangle indicating the site constraint. Bottom row shows the steady state velocity field with red indicating high speeds and blue indicating low speeds.

rely on superimposing analytical velocity deficits in turbine wakes, so the conventional wisdom is to space turbine rows as far apart as possible in the streamwise direction to give the wake a chance to recover. In contrast, the WindSE simulations are able to capture the speedup effect from streamline curvature introduced...
by the upwind turbines that concentrates the incoming flow. The WindSE layout appears to move the main turbine rows upstream such that the flow in the ‘duct’ is prevented from diverting laterally. If the turbines were placed as far downstream as possible, then the flow would have the space to expand laterally and the speedup effect would be lost.

B. Multiple Inflow Direction Optimization

In order to begin the study of multiple inflow directions, two sequential forward runs are performed with the inflow direction changing by 180 degrees and the lateral slip boundary conditions remaining constant. Both forward simulations are performed, the power outputs are summed into the total objective function, and dolfin-adjoint derives the adjoint equations based on the combined record of equation solves in both simulations. The resulting optimal layout is symmetric about both the flow centerlines and a 90 degree angle bisecting the two inflow directions, as seen in Figure 4. The layout is optimized for 32 turbines without spacing constraints and the site constraints are outlined with a black rectangle. Figure 5 shows the corresponding flow fields when the wind direction is from the left (Figure 5a) and from the right (Figure 5b).

Adding complexity, we next simulate flow from perpendicular directions. First, two simulations with a 90 degree change in inflow angle are considered. As seen in Figures 6a and 6b, two triangular ducts form to create a local speedup effect when the flow is from the north and then from the west. As the 90 degree wind direction change is subdivided into a series of smaller direction changes, the optimal layout has fewer distinct ducts but is still symmetric about the center of rotation. The magnitude of the direction changes between sequential simulations, called bins, has a strong impact on the resulting optimal layout. Depending on the size of the site constraints, the optimal layout tends to a crescent when the wind direction bins are sufficiently small. With so many evenly weighted different inflow directions it is more difficult to take advantage of a duct-induced speedup and instead the optimizer simply minimizes wake losses.

![Figure 4: Optimal layouts for 32 turbines without spacing constraints and flow from the west and east.](image)

(a) Equal weighting of inflow directions.  
(b) Westward flow is weighted 10× eastward flow.

![Figure 5: Flow fields generated by optimizing the layout over two equally weighted but 180 degrees different inflow directions.](image)

(a) The flow field for the opposite directions optimized layout when the flow is left to right.  
(b) The flow field for the opposite directions optimized layout when the flow is right to left.
(a) The flow field for the perpendicular directions optimized layout when the flow is left to right. (b) The flow field for the perpendicular directions optimized layout when the flow is top to bottom.

Figure 6: Flow fields generated by optimizing the layout over two equally weighted but 90 degrees different inflow directions.

(a) $\pi/2$ bins. (b) $\pi/4$ bins. (c) $\pi/6$ bins.

Figure 7: Optimal layouts with inflow ranging from the west to the north with varying bin direction sizes. Site constraints are marked with the black square.
C. Full Wind Rose and Axial Induction Factor Optimization

Finally, we study the optimization of turbine locations and axial induction factors using a full real world wind rose. We use the same wind rose from the Princess Amalia wind farm as the study by Fleming et al.\textsuperscript{14} The direction weights $\alpha_k$ used in the objective function are based on the 8 m/s wind rose. We optimize layouts with 6 and 12 turbines constrained to stay within a square site boundary in the center of the computational domain. The wind rose, optimal layouts, and resulting flow fields for a west wind are shown in Figures 8 and 9.

As the wind rose is refined into smaller wind direction bins the optimal layout changes to better take advantage of a few select directions that are statistically more likely. No spacing constraints were enforced in the 12 turbine optimization, and as a result WindSE placed multiple turbines close together in the northwest corner of the site. The optimal axial induction factors are also slightly lower than $1/3$ which is the optimal value for a turbine in isolation. This lower axial induction factor has the added benefit of reducing loads on the turbines.

![Wind rose with 6 bins.](image1)
![Wind rose with 12 bins.](image2)
![Wind rose with 24 bins.](image3)

![Optimal layout for 6 bins.](image4)
![Optimal layout for 12 bins.](image5)
![Optimal layout for 24 bins.](image6)

![Flow field with a west wind.](image7)
![Flow field with a west wind.](image8)
![Flow field with a west wind.](image9)

Figure 8: Layout optimization and flow field results for 6 turbines with the Princess Amalia wind rose.
Figure 9: Layout optimization and flow field results for 12 turbines with the Princess Amalia wind rose. Without spacing constraints, WindSE places multiple turbines in the northwest corner of the site.
VI. Summary and Conclusions

We have introduced a new wind farm flow model and corresponding adjoint model for wind turbine layout and axial induction factor optimization. This enables efficient gradient-based optimization of wind turbine layouts for farm-wide systems engineering analysis. We find discrete adjoints of our objective functional using the Python packages FEniCS and dolfin-adjoint, and use gradient-based optimization algorithms to find optimal layouts. The adjoint approach finds gradients at a cost that is independent of the number of design variables and facilitates the use of high fidelity flow models rather than simplified engineering models.

Our initial results for a single inflow direction demonstrate that the optimizer is able to take advantage of flow curvature and speedup effects when a dominant wind direction is present. These curvature effects are not represented by the analytical wake deficits used in common industry flow models. As a result, our optimal layouts reflect very different strategies than are common in the wind industry. Instead of spacing parallel turbine rows as far apart as possible in the streamwise direction, WindSE channels the flow toward the center of the farm and creates a local speedup. Turbines are then arranged to prevent the flow from diverting around downwind rows by moving them closer to the upwind speedup zone.

In multiple direction studies, the duct strategy is less effective for uniformly distributed wind roses. The optimal layout is tends to be symmetric about the center of rotation for optimization studies with just a few inflow directions. However, refining the wind rose binning process does affect the optimization results. Unevenly weighting the inflow directions allows the flow to recover a directional duct shape, and also reflects real world wind roses which often exhibit a preferred direction. As more evenly weighted direction bins are introduced, the directionality of a duct is less optimal and the optimizer focuses on a layout topology that minimizes wake losses. Finally, optimal axial induction factors tend to be below the optimal value for a single turbine in isolation, suggesting that turbine-level controls may be useful for power optimization and also benefit from a reduction in loads.

WindSE extends the wind farm systems engineering analysis in the NREL WISDEM toolset to include high fidelity layout and controls optimization. Ongoing work to continue development of WindSE is focused on adding greater complexity and accuracy to the flow solver and extending the simulations to three dimensions. Additional work is focused on improving the fidelity of the turbine representation to allow for more control optimization.

References


16 Gunner Chr Larsen, Helge Aagaard Madsen, Niels Troldborg, Torben J. Larsen, Pierre-Elouan Rthor, Peter Fuglsang, Sren Ott, Jakob Mann, Thomas Buhl, Morten Nielsen, and others. TOPFARM-next generation design tool for optimisation of wind farm topology and operation.


